**Numerical Modeling of Planetary Dynamos with MagIC**

By

Andrew Jon Bruneel

A senior thesis submitted to the faculty of Loyola Marymount University in partial fulfilment of the requirements for the degree of

Bachelor of Science

Department of Physics

Loyola Marymount University

May 2022

Copyright © Andrew Jon Bruneel 2022

All Rights Reserved

LOYOLA MARYMOUNT UNIVERSITY

**THESIS COMMITTEE APPROVAL**

of a thesis submitted by

Andrew Bruneel

This thesis has been read by each member of the following thesis committee and by majority vote has been found to be satisfactory.

\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date Dr. Emily K. Hawkins, Thesis Advisor

\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date Dr. Jonas Mureika, Thesis Coordinator

\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date Dr. David Berube

**ABSTRACT**

Dynamos are the internal engines that generate magnetic fields. Because of this, they have recently become one of the most popular topics of interest in the field of space physics. By learning more about dynamos, we can understand how planetary magnetic fields will evolve over time. Using this information can also aid us in the search for habitable environments across the solar system and the universe. Despite the importance of learning about dynamos, it is difficult to simulate environments where we can study the physics that occurs in planetary cores. To solve this issue, numerical models are run in tandem with laboratory experiments so that we may compare the results afterwards. By overlapping our studies, we can maximize what we are able to learn. We present the results of new numerical modeling efforts to explore details of the fluid physics involved in generating a global scale magnetic field from electrically conducting, rotating, convection using the open-source computational fluid dynamics code MagIC. These results align with accepted theory in the field, which states that dynamo conditions achieving a Magnetic Reynolds number of 40 or greater will sustain a dynamo over geological time scales. This study will examine a range of Magnetic Prandtl number values across otherwise consistent dynamo parameters, determining the necessary conditions to sustain dynamo action over time.

**CONTENTS**

1 **An Introduction to Dynamos and Numerical Modeling** . . . . . . . . . . .1

1.1 Dynamo Theory . . . . . . . . . . . . . . . . . . . . . . . . . . . 1

1.2 Motivation Behind Studying Dynamos . . . . . . . . . . . . . . . . . . 2

1.3 Dynamos Within Our Solar System . . . . . . . . . . . . . . . . . . . 2

1.4 Introduction to Numerical Dynamo Modeling . . . . . . . . . . . 4

2 **Magnetohydrodynamic Equations and Programming Specifics** . . . . . . .5

2.1 Non-Dimensional Parameters and Governing Equations . . . . . . . 5

2.2 Final Version of Governing Equations . . . . . . . . . . . . . . . 7

3 **Methods of Verification** . . . . . . . . . . . . . . . . . . . . . . . . . . . . .9

3.1 Background of Benchmark Cases . . . . . . . . . . . . . . . . . . . 9

3.2 Benchmark Case 0 . . . . . . . . . . . . . . . . . . . . . . . . . . . 9

3.3 Benchmark Case 1 . . . . . . . . . . . . . . . . . . . . . . . . . . . 11

3.4 Benchmark Case 2 . . . . . . . . . . . . . . . . . . . . . . . . . . . 13

3.5 Notes on Analysis of Results . . . . . . . . . . . . . . . . . . . . . . . 15

4 **Results of Varying the Magnetic Prandtl Number** . . . . . . . . . . . . . . 16

4.1 Original Dimensionless Parameters and Grid Parameters . . . . . . . 16

4.2 Magnetic Prandtl of 5.0 . . . . . . . . . . . . . . . . . . . . . . . 17

4.3 Magnetic Prandtl of 2.0 . . . . . . . . . . . . . . . . . . . . . . . 19

4.4 Magnetic Prandtl of 1.0 . . . . . . . . . . . . . . . . . . . . . . . 21

4.5 Magnetic Prandtl of 0.9 . . . . . . . . . . . . . . . . . . . . . . . 23

4.6 Magnetic Prandtl of 0.5 . . . . . . . . . . . . . . . . . . . . . . . 25

4.7 Summary of Results . . . . . . . . . . . . . . . . . . . . . . . . . . . 27

5 **Conclusions and Future Directions** . . . . . . . . . . . . . . . . . . . . . . .29

5.1 Discussion of Results . . . . . . . . . . . . . . . . . . . . . . . . . . . 29

5.2 Future Work . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29

**LIST OF FIGURES**

Planetary Magnetic Field Models . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

**Benchmark Cases**

Benchmark Case 0 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . 10

Benchmark Case 1 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . 12

Benchmark Case 1 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . 12

Benchmark Case 2 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . 14

Benchmark Case 2 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . 14

**Prandtl Cases**

Prandtl 5.0 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 18

Prandtl 5.0 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 18

Prandtl 2.0 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 20

Prandtl 2.0 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 20

Prandtl 1.0 Kinetic Energy . . . . . . . . . . . . . . . . . . . . . . . . . . . 22

Prandtl 1.0 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 22

Prandtl 0.9 Kinetic Energy . . . . . . . . . . . . . . . . . . . . . . . . . . . 24

Prandtl 0.9 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 24

Prandtl 0.5 Kinetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 26

Prandtl 0.5 Magnetic Energy. . . . . . . . . . . . . . . . . . . . . . . . . . . 26

**LIST OF TABLES**

Non-Dimensional Benchmark Case 0 Results . . . . . . . . . . . . . . . . . . . 10

Non-Dimensional Benchmark Case 1 Results. . . . . . . . . . . . . . . . . . . 11

Non-Dimensional Benchmark Case 2 Results. . . . . . . . . . . . . . . . . . . 13

Parameters and Grid Resolutions for Varying Magnetic Prandtl Number . . . . . . . . 16

Summary of Results from Varying Magnetic Prandtl (Dimensionless) . . . . . . . 27

**ACKNOWLEDGEMENTS**

I would like to thank Loyola Marymount University for allowing me to study and perform research at this institution, and most especially the entire physics department for supporting me throughout my thesis work. The work of other seniors and faculty truly inspired me to put forth my best work, and I am thankful for the motivation they gave me. I would also like to personally thank Dr. Emily Hawkins for taking me on as a student researcher in addition to the myriad of other responsibilities she had this academic year. Lastly, I would like to thank my family and friends for consistently supporting me throughout this process. Without their love and support, I would not have made it this far.

**CHAPTER 1: AN INTRODUCTION TO DYNAMOS AND NUMERICAL MODELING**

**1.1 Dynamo Theory**

Dynamos are what allow celestial bodies to generate and maintain magnetic fields. These systems are not present on every such body in our solar system, but Earth and several other planets have them to create global magnetic fields [8]. There is a lot left to understand about dynamo theory because it is a relatively new field, but we do know some basic information that is guiding how we are able to move forward in our studies of them [9].

It is known that to have a dynamo that generates a magnetic field, a celestial body must have a rotating, convecting, electrically conducting fluid inside its core [10]. Additionally, there must be enough kinetic energy from planetary rotation, and an internal energy source to drive convection within the fluid [8]. Generally, the heat that is required within a planet’s core for a dynamo is achieved with convection of the fluid within the outer core [11]. To discover how these magnetic fields are formed, we use magnetohydrodynamic (MHD) equations, which analyze the fluid motion over space and time that is generating the fields [5].

These equations are applicable because we alter them slightly from their original forms to be used for the fluid motion within dynamos (also assuming outer core fluid is incompressible) [5]. Although there are efforts to analyze different parameters of these MHD equations through laboratory experiments, the efforts cannot achieve planetary scale in terms of space and time. Laboratory experiments can, however, achieve more accurate conditions than computer models on local scales (as computer models fall well short of accurate parameter values) [12]. By combining our laboratory efforts with our numerical modeling, we can learn more about how the interiors of planets work.

**1.2 Motivation Behind Studying Dynamos**

Although dynamos are complex entities and have interesting physical dynamics, there is a deeper reasoning behind why we study these planetary core processes. As stated before, planetary dynamos allow celestial bodies to sustain magnetic fields. These magnetic fields create “magnetospheres” outside of planetary atmospheres that allow planets to protect themselves from radiation [3]. This radiation can be from the sun or from other plasma sources in space but shielding against it allows for habitability. Earth itself uses its magnetosphere to protect from the solar wind/radiation, and many other planets do the same. As such, dynamos can generally be seen as a requirement for life [14]. By studying dynamos throughout our solar system, we can determine the strength of magnetic fields on different bodies, and thus the level to which their magnetospheres protect from radiation. This is one very important element that needs to be explored if habitability is to be possible.

**1.3 Dynamos Within Our Solar System**

Because of the wide-ranging implications of studying dynamos, there has been extensive research on planetary magnetic fields within our solar system that are generated by dynamos. Through our findings, we know that all planets in our solar system have dynamos within their interiors except for Mars and Venus. This is evidenced by both planets’ lack of a global magnetic field (which all other planets possess). Despite lacking a dynamo today, there is evidence that Mars had one in the past, which scientists have discovered by noting magnetization within crustal remnants [13]. By looking at different planetary dynamos as well as the varying fluids inside interiors across our solar system, we can better understand how dynamos are formed or lost, and how different magnetic field strengths are achieved [6].

A picture containing chart

Description automatically generated

*Figure 1: Magnetic field models of all planets in the solar system with a dynamo* [20].

As shown in figure 1, we can see magnetic fields on each of the depicted planets with wide-ranging magnitudes on the order of millions of nanotesla. Mercury, Earth, Jupiter, and Saturn all possess dipolar dynamos (the asymmetry of Earth and Jupiter’s fields is a result of higher precision data that was collected for those planets). Uranus and Neptune, however, have definitively multipolar magnetic fields. Discovering why this is the case and how magnetic fields are different due to different conditions within their cores is the reason why scientists study dynamos. We can take physical observations from spacecraft to build numerical dynamo models that encapsulate a range of possible visuals for a planets’ dynamo (using specific parameters based on that data). Using this knowledge, we will ultimately be able to quantify the necessary components for life on planets and use them to find potential locations for life besides Earth.

**1.4 Introduction to Numerical Dynamo Modeling**

Numerical dynamo modeling is a means solving the governing equations of MHD through an iterative process. Rather than physically emulating the environment of a dynamo inside a planetary body, numerical modeling is an approach to attempting to understand how dynamos work through analysis of a simulated environment. Several codes have been created that are able to adjust input parameters in the governing equations to fit the created environments to bodies we are trying to simulate. This is achieved by adjusting dimensionless parameters that have important values such as planetary rotation built into them. The program I chose to use for simulations in this thesis work is called MagIC (an open-source code), and this allows for numerical simulations of fluid motion in a spherical shell [1].

Unlike other modeling programs such as Dedalus or Ansys that can be broadly used for different environments involving fluid dynamics, MagIC is specialized in being applied to dynamos within spherical shell systems. MagIC uses Chebyshev polynomials as well as a spherical harmonic decomposition to solve the MHD governing equations, which includes the Navier-Stokes equation [2]. The program was written in Fortran and utilizes a hybrid parallelization scheme that allows computers to distribute processing tasks among different computer cores, which decreases the time to run code. There are also several post-processing options that allow you to analyze the data further numerically (in python, for example) or even visually (in Paraview).

**CHAPTER 2: MAGNETOHYDRODYNAMIC EQUATIONS AND PROGRAMMING SPECIFICS**

**2.1 Non-Dimensional Parameters and Governing Equations**

As mentioned in the previous section, the MagIC program solves the governing equations of magnetohydrodynamics (MHD) using an iterative process, and the values returned allow us to determine if generating and sustaining a dynamo is possible with the given input parameters. If a dynamo process is possible, we will see an equilibrium of energy values once we analyze the data [15]. The governing equations of MHD -- which have been adapted to be applied to dynamos rather than space plasmas -- include the conservation of mass equation (1), Navier-Stokes equation (2), magnetic induction equation (3), and temperature evolution equation (4). Combined, they allow us to study the fluid motion within planetary cores over long time scales [16]. These equations are given as:

, (1)

, (2)

, (3)

, (4)

where [m/s] is the fluid velocity, [rad/s] is the rotation rate of the fluid, [kg/m3] is the mean density of the fluid, [kg/(ms2)] is the pressure, [1/K] is the thermal expansivity of the fluid, [m/s2] is the gravitational acceleration, () [K] is the local difference in temperature with respect to the mean fluid temperature, [m2/s] is the kinematic viscosity of the fluid, [A/ m2] is the electrical current density of the fluid, [T] is the magnetic field, [K] is the local fluid temperature, [m2/s] is the thermal diffusivity of the fluid, and [m2/s] is the magnetic diffusivity of the fluid [7]. The forces involved in (2) are, from left to right: the material rate change of momentum, the Coriolis acceleration, the pressure gradient, thermal buoyancy, centrifugal buoyancy, viscous diffusion, and the Lorentz term [7]. The above equations are examples of the dimensionalgoverning MHD equations. To reduce the complexity of our analysis on numerical dynamo models, we non-dimensionalize them. This allows us to look at ratios of the forces involved (Lorentz term, Coriolis force, *etc.*) and easily adjust their contribution to dynamo generation via non-dimensional parameters [16].

Although all equations are equally important when we are looking at numerical models, the Navier-Stokes equation (2) is the most important to understand because it contains all the non-dimensional parameters we will be adjusting during our runs of the program and will be discussed further below. These dimensionless numbers include the Rayleigh number (*Ra*), Ekman number (*E*), Prandtl number (*Pr*), and the Magnetic Prandtl number (*Pm*) [2]. These numbers are the ratio of terms in equation (2) that are important to the generation of dynamos such as viscous diffusion, convection, thermal diffusion, magnetic diffusion, and dimensions of the planet’s fluid layer, for example. These parameters are given by the following equations:

, (5)

, (6)

, (7)

. (8)

By grouping these factors together into named parameters, it allows us to write the governing equations in a simpler fashion and makes it much easier to edit code when we want to see the effect that changing certain aspects of dynamos will have on the simulation. To have a self-sustaining dynamo, flow inertia must be able to overcome certain elements such as viscous and thermal diffusion. These parameters cause fluid motion to be decreased, which in turn lowers the strength of the planetary magnetic field that a given dynamo creates [3]. By utilizing the Ekman, Rayleigh, Prandtl, and Magnetic Prandtl numbers, we can group related factors together into non-dimensional parameters that allow us to adjust specific variables within our models. For example, the Prandtl number is a ratio of the kinematic viscosity to the thermal diffusivity. This number relates to the type of fluid within a planetary core. Changing our Magnetic Prandtl number allows us to create numerical models according to different fluids that may be within the core.

**2.2 Final version of Governing Equations**

Although there can be slight variation between governing equations depending on how the non-dimensional parameters are written, all of them will be equivalent. Equation (5) below shows how the Navier-Stokes equation is written once it has been configured using only non-dimensional parameters [16]. All the governing equations will change once you do this, but the Navier-Stokes equation contains all the parameters that will be affected and is thus a good example to use:

. (9)

The prime (‘) symbol next to all the variables indicates that they are dimensionless. Once the governing equations have been written like this, the control parameters (Ekman, Rayleigh, Prandtl, Magnetic Prandtl) are easily edited to apply models to different planetary bodies. It is important to note that even the most extreme numerical dynamo simulations still fall several orders of magnitude short in terms of simulating the exact environments of dynamos (each non-dimensional parameter remains small compared to actual measured values). To aid in solving this problem, the control parameters are edited proportionately so we can learn from the models that we produce and ultimately scale model results up to planetary conditions.

**CHAPTER 3: METHODS OF VERIFICATION**

**3.1 Background of Benchmark Cases**

To begin modifying dimensional parameters within the governing equations, I first had to verify that the software being used for this numerical experimentation can produce accurate results. To begin, I ran several “benchmark” cases out of a verified paper in the dynamo modeling field [2]. This paper, titled “A Numerical Dynamo Benchmark” by Christensen *et al.*, uses a different modeling software for numerical dynamos than MagIC. If MagIC sufficiently reproduces the results of this paper by adjusting the input parameters to match what was used in the cases, then it can be used to perform new runs on varying parameters with confidence that the outputs will be accurate [1]. The main parameters that will be adjusted in these cases are the grid parameters (which have an implication on the resolution of the run), as well as the non-dimensional parameters (*i.e*. the Rayleigh, Ekman, Prandtl, and Magnetic Prandtl numbers). The benchmark cases cover a variety of parameters that could be seen in a real, physical, dynamo which ensures that the software is working correctly across all parameters. After repeating the accepted results in the field, we can move forward and begin to adjust the same parameters we tested from the benchmark cases. By adjusting the Magnetic Prandtl, we will be able to see the impact of fluid properties on the ability of a dynamo to form.

**3.2 Benchmark Case 0**

This case involved a non-magnetic core with rotating convection. Because the fluid within this core was not electrically conducting, this benchmark case is not classified as a dynamo [10]. This means that there should not be a magnetic energy associated with this case, although a kinetic energy should be measurable [2]. This ensures that one of the simpler cases for MagIC to handle is running properly and returning dimensionless energy values that prove convection is occurring within the core.

|  |  |
| --- | --- |
| Accepted Kinetic Energy | 58.3481 ± .050 |
| Measured Kinetic Energy | 58.3481 |

*Table 1: Non-Dimensional Kinetic Energy Results of Benchmark Case 0*

**Chart

Description automatically generated with medium confidence**

*Figure 2: Benchmark Case 0 Graph of Kinetic Energy Over Viscous Diffusion Times*

This case returns results that are consistent with what we see from the Christensen study [2]. It is noteworthy that the graph is broken into two axes – poloidal (“pol”) and toroidal (“tor”). In spherical units, the poloidal axis accounts for the kinetic energy in the *r* and axes, while the toroidal energy accounts for the axis energy. These are calculated separately due to how MagIC iteratively solves the governing equations of MHD using Chebyshev polynomials in the radial direction and spherical harmonic decomposition in the azimuthal direction [1]. The graphs utilized to analyze the equilibrium of energy values are created from functions written in MagIC that post-process the data. It is significant to point out that both the x-axis (Kinetic Energy) and y-axis (Time) are non-dimensional and therefore do not have units, due to being products of the non-dimensional Navier-Stokes equation discussed previously. The time is based on viscous diffusion times – the time for the fluid to viscously diffuse throughout the fluid layer. If the non-dimensional energy can equilibrate across viscous diffusion times, then we can infer that a planetary dynamo with equivalent parameters will be able to do the same across geological time scales (by scaling our results).

**3.3 Benchmark Case 1**

This case involved a dynamo with an insulating inner core co-rotating with the outer boundary. This means that there is a dynamo, and it is expected that there will be both a kinetic energy and magnetic energy once the dynamo has stabilized in the simulation [2]. Knowing that the inner core is “co-rotating” with the outer boundary tells us that both the inner core and outer core are moving about the rotation axis at the same rate.

|  |  |
| --- | --- |
| Accepted Kinetic Energy | 30.7714 ± .020 |
| Measured Kinetic Energy | 30.7719 |
| Accepted Magnetic Energy | 626.406 ± .020 |
| Measured Magnetic Energy | 626.401 |

*Table 2: Non-Dimensional Energy Results of Benchmark Case 1*

Chart

Description automatically generated

*Figure 3: Benchmark Case 1 Graph of Kinetic Energy Over Viscous Diffusion Times*

Chart, line chart

Description automatically generated

*Figure 4: Benchmark Case 1 Graph of Magnetic Energy Over Viscous Diffusion Times*

Once again, our graphs correctly predict that we see a dynamo self-sustain over time. Although we are seeing lower numbers for our kinetic and magnetic energy values in our tables, it is because we normalize across shell volume for those values. This is because the graphs output from MagIC for energies are not normalized. However, Christensen normalizes his values, and thus we need to do the same to verify ours.

**3.4 Benchmark Case 2**

Benchmark case 2 simulates a dynamo with a conducting and super-rotating inner core. This case is like benchmark case 1 in that we also predict a dynamo will be maintained across viscous diffusion times, but the environment is much different. The inner core is conducting rather than insulating and describing it as “rotating” rather than “co-rotating” means the inner core is rotating at a faster rate about the axis than the outer core.

|  |  |
| --- | --- |
| Accepted Kinetic Energy | 42.388 .050 |
| Measured Kinetic Energy | 42.3652 |
| Accepted Magnetic Energy | 845.60 .04 |
| Measured Magnetic Energy | 847.4835 |

*Table 3: Non-Dimensional Energy Results of Benchmark Case 2*

Chart

Description automatically generated

*Figure 5: Benchmark Case 2 Graph of Kinetic Energy Over Viscous Diffusion Times* Chart, line chart

Description automatically generated

*Figure 6: Benchmark Case 2 Graph of Magnetic Energy Over Viscous Diffusion Times*

This last case is a final check to ensure that all benchmark cases are running at the correct parameters. Although our kinetic energy falls within the accepted range of error, our magnetic energy is outside of it. This is because Christensen *et al.* uses a different program to calculate his energies for the benchmark cases than MagIC [2]. When I replicated the grid parameters from the Christensen study, I was unable to reproduce their results using MagIC. However, increasing the resolution of my run allowed me to get the same results as Christensen *et al.* Moving forward, I found that I needed slightly higher grid parameters to achieve accurate results for my energy, and I made changes accordingly as I varied the Magnetic Prandtl number for my future runs.

**3.5 Notes on Analysis of Results**

Once the results from the above simulations were compiled, they had to be analyzed to ensure that everything was working correctly. As mentioned previously, by comparing the accepted and measured values for kinetic energies and magnetic energies, we can determine if our results are within the error range and are therefore acceptable. If the results are within the error range, that means the final values of kinetic and magnetic energy are correct. These are the values that we get once the dynamo has stabilized and reached a self-sustaining point. If the dynamo does not reach this point, energy values will plummet.

**CHAPTER 4: RESULTS OF VARYING THE MAGNETIC PRANDTL NUMBER**

**4.1 Original Dimensionless Parameters and Grid Parameters**

To compare results between cases with varying Magnetic Prandtl numbers, there must be a baseline control of parameters that influence dynamo generation. This includes the Ekman, Rayleigh, and Prandtl numbers, which were chosen to be fixed values under a case study from Soderlund *et al.* [3]. A table of parameter and grid resolution values for each case examined in this thesis is listed below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Magnetic Prandtl | Prandtl | Rayleigh | Ekman | nrmax | nphitot |
| 5.0 | 1.0 | 1.42 106 | 1 10-4 | 41 | 192 |
| 2.0 | 1.0 | 1.42 106 | 1 10-4 | 41 | 192 |
| 1.0 | 1.0 | 1.42 106 | 1 10-4 | 41 | 192 |
| 0.9 | 1.0 | 1.42 106 | 1 10-4 | 61 | 288 |
| 0.5 | 1.0 | 1.42 106 | 1 10-4 | 61 | 288 |

*Table 4: Parameters and Grid Resolutions for Varying Magnetic Prandtl Number*

All non-dimensional numbers (excluding the Magnetic Prandtl number) have been fixed at set values that we know will support a dynamo across viscous diffusion times (given a high enough Magnetic Prandtl number). This was proven by Soderlund *et al.* at a Magnetic Prandtl number of 2.0, which is the reference case we are using as a baseline for the parameters applied to our other runs [3].

In addition to non-dimensional numbers, I have listed the grid resolutions used for each run. These have no affect on the run itself (besides the amount of wall clock time the computer will take to complete it) but will increase the accuracy of our results. Essentially, we are sacrificing efficiency for a higher confidence in our results. The grid parameters are as follows: nrmax [integer value], which is the number of grid points in the radial direction in the outer core, and nphitot [integer value], the number of grid points in the azimuthal direction. The grid parameters used for the Magnetic Prandtl numbers 5.0, 2.0, and 1.0 are matched with the parameters from the original Soderlund *et al.* case [3]. However, as this number decreases, we are required to increase our grid parameters to maintain accuracy across our cases, which is the reason for the scaled-up resolutions for Magnetic Prandtl 0.9 and 0.5 [12].

**4.2 Magnetic Prandtl of 5.0**

Our first case to analyze is on the highest end of our range of values for the Magnetic Prandtl number. The goal of varying this number across runs is to find a point where the parameter is too low for a dynamo to self-sustain across viscous diffusion times. Starting with a high number for Magnetic Prandtl provides us with an inference that we will begin with stronger dynamos and progress towards dynamos that die out very quickly at the lower Magnetic Prandtl values.

Chart

Description automatically generated

*Figure 7: Kinetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 5.0*

Chart, histogram

Description automatically generated

*Figure 8: Magnetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 5.0*

We can see from our graphs of kinetic and magnetic energy that we do reach an equilibrium across time from this simulation. This means that our dynamo has the necessary convection and electrical conduction to create a long-lasting planetary magnetic field. Because the simulations have much larger input parameters, we do not see the typical flat-line stabilization of energy values that we found in the Christensen *et al.* benchmark cases [2]. A key difference from our analysis of the benchmark cases compared to current runs varying Magnetic Prandtl is that the graphs from the current runs were created outside of MagIC. Using python libraries, I plotted these results so that I could superimpose all magnetic energies onto the same graph. This way, the data is condensed, and it is easier to see the dynamo action. Additionally, due to the innately larger energy values we see in these simulations, I scaled the y-axis logarithmically to easily see the energy across time. A table summarizing the numerical values of all novel runs in this thesis is provided below in Section 4.7.

**4.3 Magnetic Prandtl of 2.0**

The simulation with a Magnetic Prandtl value of 2.0 is the only instance we have where we are directly comparing our results to a published paper. This case has the same Ekman, Rayleigh, Prandtl, and Magnetic Prandtl numbers as a Soderlund *et al.* case [3]. This way, results found here can are ensured to be consistent, therefore increasing the confidence of results from other runs. As found in the Soderlund *et al.* case, these parameters should sustain a dynamo.

**Chart

Description automatically generated**

*Figure 9: Kinetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 2.0*

**Chart, histogram

Description automatically generated**

*Figure 10: Magnetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 2.0*

Looking at the results from this simulation, we do in fact see a dynamo sustained. I was unable to pinpoint why we see the flat-line of both kinetic and magnetic energy across the first 4 viscous diffusion times, but this does not affect the final results we get as we note they do stabilize. Comparing the final equilibrated numbers to Soderlund *et al.*, I found they are equivalent. Because Soderlund *et al.* reports the Nusselt and Reynolds numbers, I used those to compare my results with the paper. MagIC has a built-in function to report the Nusselt numbers, and the Reynolds number can be found using the kinetic energy. This equation is given below as:

, (10)

is the total kinetic energy (not normalized by shell volume). After I was satisfied that these numbers aligned with the results from the paper, I felt more confident in my results for the other cases. As the Magnetic Prandtl decreased throughout the first two runs, the magnetic energy on the y-axis dips below a value of 103 at its lowest value (compared to a higher value from Pm = 5.0) and will continue to die out as we stray further from ideal dynamo conditions.

**4.4 Magnetic Prandtl of 1.0**

As our Magnetic Prandtl continues to get smaller, we anticipate that the simulated dynamo will die out across viscous diffusion times. It is noteworthy that so far, our poloidal and toroidal kinetic energy have remained consistent throughout simulations. This means we have well-controlled parameters for our runs since we want the only limiting factor on dynamo generation to be the dynamo’s ability to sustain its magnetic energy (given that we are varying only the Magnetic Prandtl number). This simulation with Pm = 1.0 was the last simulation ran with the same grid parameters as the Soderlund *et al.* case, which also uses MagIC [3]. The future cases with lower Magnetic Prandtl required higher grid resolution to be resolved with accuracy due to the increase in magnetic diffusivity.

**Chart

Description automatically generated**

*Figure 11: Kinetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 1.0*

**Chart, histogram

Description automatically generated**

*Figure 12: Magnetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 1.0*

Similarly to our previous runs with a higher Magnetic Prandtl number, we find a dynamo that is sustained across all viscous diffusion times. The trend of decreasing magnetic energy continues, most notably with the inner core toroidal magnetic energy that we see encroaching on the x-axis when compared to previous simulations. Because all graphs for the Magnetic Prandtl varied runs are scaled logarithmically, the distance to zero magnetic energy is closer than it appears. After viewing this result, I expected to find a dynamo that died out from one of the following runs where the Magnetic Prandtl number will be lowered further.

**4.5 Magnetic Prandtl of 0.9**

Due to the results from the Magnetic Prandtl of 1.0, I expected the dynamo to be much weaker from this run of MagIC. As mentioned previously, the grid parameters were raised for this simulation to accurately depict the fluid motion occurring within the outer core. Although kinetic energy has remained largely consistent throughout simulations, it has been increasing slightly as the Magnetic Prandtl gets lower. This means we have a higher Reynolds number, consequently lowering the number of viscous diffusion times that MagIC needs to simulate before verifying the existence of a dynamo.

**Graphical user interface, chart

Description automatically generated**

*Figure 13: Kinetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 0.9*

**Chart, histogram

Description automatically generated**

*Figure 14: Magnetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 0.9*

This run is the first instance of our dynamo completely dying out. This occurs right around the sixth viscous diffusion, where we see all magnetic energies across the poloidal and toroidal axes approach a value of zero. Despite the electrical conduction not being large enough to sustain our dynamo over time, the convection has remained consistent (visualized with our stable kinetic energy values on the graph). This means that the Magnetic Prandtl has only affected our magnetic energies, which is a good sign and indicates that our results are consistent with the theory. As we simulate Pm = 0.5, we expect to see another dynamo that dies out across viscous diffusion times, this time much more quickly.

**4.6 Magnetic Prandtl of 0.5**

This is the last dynamo simulation being tested across our range of Magnetic Prandtl values. Based on the results we found from Pm = 0.9, we expect to see another dynamo that cannot self-sustain across viscous diffusion times. This simulation was conducted on equivalent grid resolution to the Magnetic Prandtl 0.9 case due to a higher magnetic diffusivity than the previous cases.

**Graphical user interface

Description automatically generated**

*Figure 15: Kinetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 0.5*

**Chart

Description automatically generated**

*Figure 16: Magnetic Energy Over Viscous Diffusion Times for Magnetic Prandtl of 0.5*

As expected, our dynamo dies out (much quicker than our case of Pm = 0.9). Before the second viscous diffusion, all magnetic energy is lost. This confirms the limit we found from our prior simulation, showing that with the given conditions of these simulations, we must have a Magnetic Prandtl of 1.0 or greater to sustain a dynamo for the conditions simulated. We also see another increase in the kinetic energy (convection) occurring within the core, but without the requisite electrical conduction, it is not enough to see the dynamo last. Below, we will look at a table of values collected through post-processing analysis of all simulations. This will aid in understanding the trends that can be found from the data collected.

**4.7 Summary of Results**

Below is the summary of results from all simulation trials. Included are the Magnetic Reynolds number, total kinetic energy (normalized over shell volume), outer/inner core magnetic energy (normalized over shell volume), and the viscous diffusion times required for the simulation.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Magnetic Prandtl | Magnetic Reynolds | Total Kinetic Energy (normalized) | Outer Core Magnetic Energy (normalized) | Inner Core Magnetic Energy (normalized) | Viscous Diffusion Times |
| 0.5 | 20.7679 | 862.6161 | 0 | 0 | 7 |
| 0.9 | 38.3213 | 906.4963 | 0.0024 | 0.0001 | 7 |
| 1 | 39.8561 | 794.2544 | 1337.8683 | 79.4277 | 15 |
| 2 | 69.5122 | 603.9923 | 3268.5114 | 182.1650 | 13 |
| 5 | 154.4430 | 477.0536 | 6269.8652 | 166.2364 | 5 |

*Table 5: Summary of Results from Varying the Magnetic Prandtl Number (Dimensionless)*

Table 4 shows all of the results from the calculations performed on data output from MagIC. The magnetic energy from the lowest Magnetic Prandtl simulations is essentially zero, confirming that a dynamo will not be sustained with Pm < 1.0 using the parameters simulated. These results can be checked for accuracy against their Magnetic Reynolds numbers. A Magnetic Reynolds number of signifies that there will be a dynamo, given the equation [15]. Essentially, the Magnetic Reynolds number compiles the convection and electrical conduction values into a singular number. If the previous values are high enough, you will have a Magnetic Reynolds number higher than 40, meaning you can sustain a dynamo.

**CHAPTER 5: CONCLUSIONS AND FUTURE DIRECTIONS**

**5.1 Discussion of Results**

The results from this study have shown us valuable trends about dynamo formation given a varying Magnetic Prandtl number. With the parameters listed that were used for the varying Magnetic Prandtl number cases, we have benchmark cases we can verify against existing dynamos. If we proportionately scale-up our input and output parameters to be comparable to actual values for observed dynamos, we can confirm the results from this study. Once these results are checked, we can apply the limits of Magnetic Prandtl to new planets that are discovered. We can determine if a given planet will have a dynamo (and be able to sustain it across geological time scales) by observing parameters such as the Ekman, Rayleigh, Prandtl, and Magnetic Prandtl numbers and using studies like this to compare those input parameters to dynamo simulations. This makes it much easier to determine the potential life for planets and exoplanets with magnetic fields we know of already, as well as planets whose existence is unknown as of right now.

**5.2 Future Work**

Moving forward, similar studies can be conducted varying different parameters to measure their impact on dynamo generation. With a larger library of results to pull from, we will be able to easily classify new celestial bodies as having dynamos or not and improve the speed at which we can determine the potential for life on these bodies. Future iterations of this study would begin to use the results from this paper and measure them against planets with similar scalable parameters. By comparing them to known planets, we can see if dynamos are accurately predicted by this study. Once these results are confirmed, adding the variation of more parameters to the study will allow us to then compare those results to other planets as well. In conclusion, we can determine that the results from varying the Magnetic Prandtl number are accurate in terms of aligning with the theory, and untested with actual observed data. The predicted dynamo “cut-off” of a Magnetic Reynolds number of 40 proved to be accurate based on the data collected from these simulations, which means they are likely accurate and within the expected range of error for results.

**BIBLIOGRAPHY**

[1] <https://magic-sph.github.io/> (accessed on 3 October, 2021)

[2] Christensen, U.R., *et al.*, “A numerical dynamo benchmark” *Physics of the Earth and Planetary Interiors* **128**, 25-34 (2001)

[3] Soderlund, K., King, E., Jonathan, A., “The influence of magnetic fields in planetary dynamo models”, *Earth and Planetary Science Letters* **392**, 333-334 (2012)

[4] Schaeffer, N., Cardin, P., “Quasi-geostrophic kinematic dynamos at low magnetic Prandtl number”, *Earth and Planetary Science Letters*, **245**, Pages 565-604 (2006)

[5] Glatzmaier, G., *Introduction to Modeling Convection in Planets and Stars*, Princeton University Press (2014)

[6] Philidet, J., *et al.*, “Magnetohydrodynamics of stably stratified regions in planets and stars”, *Geophysical and Astrophysical Fluid Dynamics*, **114**, 336-355 (2019)

[7] Hawkins, E. “Experimental Investigations of Convective Turbulence in Planetary Cores”, ProQuest #28001070 (2020)

[8]<https://web.archive.org/web/20150118213104/http://www.usgs.gov/faq/?q=categories%2F9782%2F2738> (accessed on 13 November, 2021)

[9] Roberts, P. H., “Theory of the Geodynamo”, *Treatise of Geophysics (ScienceDirect)*, **8**, 57-90 (2015)

[10] Vazquez, M., Palle, E., Montanes Rodriguez, P., “The Earth as a Distant Planet: A Rosetta Stone for the Search of Earth-like Worlds”, *Astronomy and Astrophysics Library*, 316-317 (2010)

[11] Olson, P., “The geodynamo’s unique longevity”, *Physics Today*, **66**, 11, 30 (2013)

[12] Christensen, U. R., Wicht, J., “8.08 - Numerical Dynamo Simulations”, *Treatise on Geophysics*, **8**, 245-282 (2007)

[13] Roberts, J.H., Lillis, R., J., Manga, M., “Giant Impacts on early Mars and the cessation of the Martian dynamo”, *Journal of Geophysical Journal*, **114** (2009)

[14] Dehant, V., *et al*., “Planetary Magnetic Dynamo Effect on Atmospheric Protection of Early Earth and Mars”, *Space Sci Rev*, **129**, 279-300 (2007)

[15] Roberts, P., Glatzmeier, G., “The Geodynamo, Past, Present and Future”, *Geophys. Astrophys. Fluid Dynamics*, **94**, 47-84 (2000)

[16]Mattheij, R.M.M., Rienstra, S.W., Thije Boonkkamp, J.H.M., "§7.4 – Scaling and Reduction of the Navier–Stokes Equations". Partial Differential Equations: Modeling, Analysis, Computation. SIAM,148–155 (2005)

[17] Wicht, J., “Inner-core Conductivity in numerical dynamo simulations”, Physics of the Earth and Planetary Interiors, **132**, 281-302 (2002)

[18] Gastine, T., Wicht, J., “Effects of compressibility on driving zonal flow in gas giants”, Icarus, **219**, 428-442 (2012)

[19] Schaeffer, N., “Efficient spherical harmonic transforms aimed at pseudospectral numerical simulations”, Geochemistry, Geophysics, Geosystems, **14**, 751-758 (2013)

[20] Cao, H., et al. “A dynamo explanation for Mercury’s anomalous magnetic field”, Geophysical Research Letters, **41**, 4127-4134 (2014)